GCE

Further Mathematics B MEI

Y420/01: Core Pure

A Level

Mark Scheme for June 2022

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. ‘can’t do’, ‘don’t know’)
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn’t an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads “3 s.f”

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

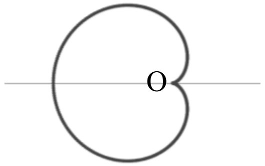
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold “In this question you must show detailed reasoning”, or the command words “Show” and “Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

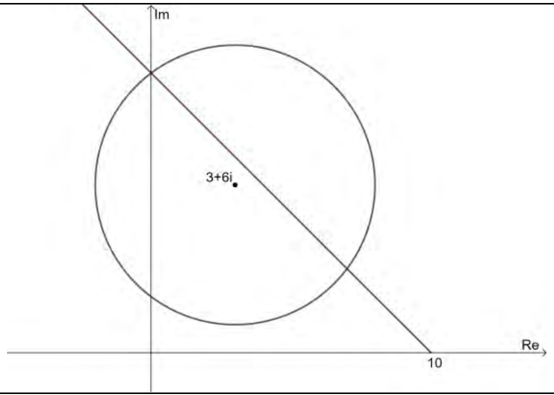
Question			Answer	Marks	AOs	Guidance
1	(a)		$(r+1)^3 - r^3 = r^3 + 3r^2 + 3r + 1 - r^3$ $= 3r^2 + 3r + 1$	B1	1.1	Soi by M1A1
			$\sum_{r=1}^{n}(3r^2 + 3r + 1) = \sum_{r=1}^{n}[(r+1)^3 - r^3]$ $= 2^3 - 1^3 + 3^3 - 2^3 + \cdots + n^3 - (n-1)^3 + (n+1)^3 - n^3$	M1	2.5	Cannot use standard summation formulae for final 2 marks
			$= (n+1)^3 - 1$	A1 [3]	2.2a	isw
1	(b)		$3\sum_{r=1}^{n} r(r+1) + n = (n+1)^3 - 1$	M1*	2.5	$\sum_{r=1}^{n} 1 = n$used and starting to rearrange
			$\Rightarrow \sum_{r=1}^{n} r(r+1) = \frac{1}{3}[(n+1)^3 - 1 - n]$	A1	3.1a	
			$= \frac{1}{3}(n+1)[(n+1)^2 - 1]$	M1dep*	1.1	Factorising with n or n + 1 correctly
			$= \frac{1}{3}n(n+1)(n+2)$	A1 [4]	2.2a	Allow **SC2** for correct solution using standard summation formulae.

2			**DR** $\int_3^k \frac{1}{x^2 - 4x + 5}\,dx = \int_3^k \frac{1}{(x-2)^2 + 1}\,dx$	M1	3.1a	completing the square, (ignore limits)
			$= [\arctan(x-2)]_3^k$	A1	1.1	$[\arctan(x-2)]$ or $[\arctan u]$ if $u = x - 2$ (ignore limits)
			$\lim_{k\to\infty}[\arctan(k-2)] = \frac{\pi}{2}$	E1	2.4	Clear limit argument
			$= \arctan(k-2) - \frac{\pi}{4}$or $\frac{\pi}{2} - \frac{\pi}{4}$	B1	1.1	
			$\Rightarrow$ integral $= \frac{\pi}{4}$	B1 [5]	2.2a	

Question			Answer	Marks	AOs	Guidance
3			**DR**			
			$3\cosh x = 2\sinh^2 x = 2(\cosh^2 x - 1)$	M1	3.1a	$\sinh^2 x = \cosh^2 x - 1$ used
			$\Rightarrow 2\cosh^2 x - 3\cosh x - 2 = 0$	A1	1.1	
			$\Rightarrow (2\cosh x + 1)(\cosh x - 2) = 0$	M1	1.1	Solve their quadratic
			$\Rightarrow \cosh x = -\frac{1}{2}$ or 2	A1	1.1	Don't need to see –½ if factorisation or quadratic formula shown
			$\Rightarrow x = \ln(2+\sqrt{3})$	A1	1.1	
			or $-\ln(2+\sqrt{3})$	A1	1.1	or $\ln(2-\sqrt{3})$
				[6]		
4	(a)		$\begin{vmatrix} m & 2 & 1 \\ 0 & 1 & -2 \\ 2 & 0 & 3 \end{vmatrix} = 3m - 2\times 4 + 1\times(-2) = 3m - 10$	M1	3.1a	finding determinant
				A1	1.1	
			so $3m - 10 = 0 \Rightarrow m = \frac{10}{3}$	A1	1.1	
			Alternative solution			
			$\begin{pmatrix} m \\ 0 \\ 2 \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} m \\ 0 \\ 2 \end{pmatrix} . \begin{pmatrix} 3 \\ -6 \\ -5 \end{pmatrix}$	M1	3.1a	
			$= 3m - 10\ [= 0]$	A1	1.1	
			$\Rightarrow m = \frac{10}{3}$	A1	1.1	
				[3]		
4	(b)		$\det \boldsymbol{M} = 4k + 3$	B1	1.2	
			$\det(\boldsymbol{NM}) = \det \boldsymbol{N} \times \det \boldsymbol{M}$	M1	3.1a	Soi
			$\Rightarrow (3k+1)(4k+3) = 2$	M1	1.1	Equating to 2
			$k = -1$ or $-\frac{1}{12}$	A1	1.1	
			Alternative solution			
			$\det(NM) = (ak-3b)(c+4d) - (a+4b)(ck-3d)$	B1	1.2	
			$(ak-3b)(c+4d) - (a+4b)(ck-3d) = 2$	M1	3.1a	
			$(3k+1)(4k+3) = 2$	M1	1.1	
			$k = -1$ or $-\frac{1}{12}$	A1	1.1	
				[4]		

Question			Answer	Marks	AOs	Guidance
5	(a)			M1 A1 [2]	1.1 1.1	symmetrical loop about the initial line correct shape with cusp at O
5	(b)		$A = \int_0^{2\pi} \frac{1}{2} a^2 (1-\cos\theta)^2 \, \mathrm{d}\theta$	M1	1.1a	correct integral and limits, condone missing $d\theta$ limits can be soi by later work may see $\int_0^{\pi} a^2(1-\cos\theta)^2 \, d\theta$
			$= \frac{1}{2} a^2 \int_0^{2\pi} \left(1 - 2\cos\theta + \frac{1}{2}[1+\cos 2\theta]\right) \mathrm{d}\theta$	M1 M1	1.1 3.1a	Expanding correctly substituting for $\cos^2\theta$
			$= \frac{1}{4} a^2 \left[3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi}$	B1	1.1	$k\left[3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta\right]$
			$= \frac{3}{2}\pi a^2$	A1cao [5]	1.1	
6			$\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1-2 & 1 \end{pmatrix}$ so true when $n = 1$	B1	2.1	
			[Assume true for $n = k$]			
			$\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2^k & 0 \\ 1-2^k & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$	M1	2.1	
			$= \begin{pmatrix} 2^{k+1} & 0 \\ 2 - 2^{k+1} - 1 & 1 \end{pmatrix}$	M1	2.1	Intermediate step seen
			$= \begin{pmatrix} 2^{k+1} & 0 \\ 1 - 2^{k+1} & 1 \end{pmatrix}$ [so true for $n = k + 1$]	A1	2.3	
			As true for $n = 1$, and if true for $n = k$ then true for $n = k + 1$, true for all n	B1cao [5]	2.4	Must receive all 4 previous marks for this to be awarded

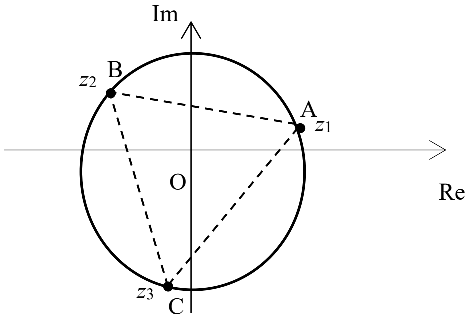
Question			Answer	Marks	AOs	Guidance
7			$\frac{x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$	M1	2.1	correct partial fractions
			$\Rightarrow x+1 = A(x^2+1) + (Bx+C)(x-1)$			
			$x = 1 \Rightarrow 2 = 2A \Rightarrow A = 1$	A1	1.1	
			coefficient of x^2: $0 = A + B \Rightarrow B = -1$	A1	2.1	
			constants: $1 = A - C \Rightarrow C = 0$	A1	2.1	
			$\int_2^3 \frac{x+1}{(x-1)(x^2+1)}\,\mathrm{d}x = \int_2^3 \left(\frac{1}{x-1} - \frac{x}{x^2+1}\right)\mathrm{d}x$			
			$= \left[\ln(x-1) - \frac{1}{2}ln(x^2+1)\right]_2^3$	B1ft	2.1	$\ln(x-1)$
				M1	2.1	$k\ln(x^2+1)$ or $u = x^2+1 \Rightarrow \int \frac{1}{2u}\,\mathrm{d}u$
				A1ft	1.1	$k = -\frac{1}{2}$ or $-\frac{1}{2}\ln u$
			$= \ln 2 - \frac{1}{2}\ln 1 0 + \frac{1}{2}\ln 5\ [-\ln 1]$			
			$= \ln 2 - \frac{1}{2}\ln\frac{10}{5}$	M1	1.1	Combining two of their logarithm terms correctly
			$= \ln 2 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln 2$	A1 [9]	2.2a	AG

Question			Answer	Marks	AOs	Guidance
8	(a)			M1 A1 B1 B1 [4]	1.1 1.1 1.1 1.1	half line from 10 at 45° to Real axis implied by angle shown or meeting the Imaginary axis at 10 circle centre $3 + 6i$ circle meeting half line on Imaginary axis
8	(b)		**DR** one point is 10i $k^2 = (3-0)^2 + (6-10)^2$ $\Rightarrow k = 5$ line $x + y = 10$ $(x-3)^2 + (10-x-6)^2 = 25$ $\Rightarrow 2x^2 - 14x = 0$ $\Rightarrow x = 7, \quad y = 3$ other point is $7 + 3i$	B1 M1* A1 M1dep* M1 A1 A1	1.1 3.1a 1.1 3.1a 1.1 1.1 3.2a	 soi solving $x + y = 10$ and circle equation simultaneously Rearranging into a quadratic = 0 Could see solutions as $\sqrt{58}(\cos 0.405 + i \sin 0.405)$or $\sqrt{58}e^{0.405i}$
			Alternative solution one point is 10i eqn of perp from (3, 6) to chord is $y = x + 3$ solving with $x + y = 10$ midpoint of chord is $\left(3\frac{1}{2}, \quad 6\frac{1}{2}\right)$ other end of chord is $\left(2 \times 3\frac{1}{2} - 0, \quad 2 \times 6\frac{1}{2} - 10\right)$ ⇒ other point of intersection is (7, 3) this represents $7 + 3i$	 B1 M1 M1 A1 M1 A1 A1		 oe, e.g. midpoint of chord has equal x and y displacements from (3, 6) or by inspection oe Could see solutions as $\sqrt{58}(\cos 0.405 + i \sin 0.405)$or $\sqrt{58}e^{0.405i}$
				[7]		

Question			Answer	Marks	AOs	Guidance
9	(a)		$\sinh k > -1$ $\Rightarrow k > \sinh^{-1}(-1) = \ln\left(-1+\sqrt{2}\right)$	M1 A1 [2]	2.1 2.2a	May be in exponential form **AG**
9	(b)	(i)	$f'(x) = \dfrac{\cosh x}{1+\sinh x}$	M1 A1 [2]	1.1 1.1	chain rule
9	(b)	(ii)	$f''(x) = \dfrac{(1+\sinh x)\sinh x - \cosh^2 x}{(1+\sinh x)^2}$ $f''(x) = \dfrac{\sinh x - 1}{(1+\sinh x)^2}$	M1 M1 A1 [3]	1.1 3.1a 1.1	quotient or product rule $cosh^2 x - sinh^2 x = 1$ used
9	(c)		$f(0) = 0, \quad f'(0) = 1, \quad f''(0) = -1$ $f(x) = x - \dfrac{1}{2}x^2$	B1ft M1 A1cao [3]	1.1 1.1 1.1	soi Maclaurin expansion attempted, must see their values substituted in
9	(d)		$\dfrac{\ln(1+\sinh(0.1)) - 0.095}{\ln(1+\sinh(0.1))} \times 100$ = 0.48%	M1 A1 [2]	1.1 1.1	Allow –0.48%

Question			Answer	Marks	AOs	Guidance
10	(a)		$\alpha \times \frac{2}{\alpha} \times \beta \times 3\beta = \frac{6}{4}$ $\Rightarrow \beta = -\frac{1}{2}$	M1 A1	3.1a 1.1	
			$\alpha + \frac{2}{\alpha} + \beta + 3\beta = -\frac{16}{4}$ $\Rightarrow \alpha^2 + 2\alpha + 2 = 0$	M1 A1	3.1a 1.1	`
			$\alpha = \frac{-2 \pm \sqrt{4-8}}{2} = -1+i, \quad -1-i$	M1	1.1	Solving their quadratic to find α
			so roots are $-1+i, \quad -1-i, \quad -\frac{1}{2}, \quad -\frac{3}{2}$	A1 [6]	3.2a	
10	(b)		$(-1+i)(-1-i) + (-1+i)\left(-\frac{1}{2}\right) + (-1+i)\left(-\frac{3}{2}\right)$ $+(-1-i)\left(-\frac{1}{2}\right) + (-1-i)\left(-\frac{3}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{a}{4}$	M1	1.1	Allow in terms of α and β, no missing terms
			$\Rightarrow a = 27$	A1	1.1	
			$(-1+i)(-1-i)\left(-\frac{1}{2}\right) + (-1+i)(-1-i)\left(-\frac{3}{2}\right)$ $+(-1+i)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) + (-1-i)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) = -\frac{b}{4}$	M1	1.1	Allow in terms of α and β, no missing terms
			$\Rightarrow b = 22$	A1	1.1	
			Alternative solution $(x+1+i)(x+1-i)(2x+1)(2x+3) = 0$ $\Rightarrow (x^2+2x+2)(4x^2+8x+3) = 0$ $\Rightarrow 4x^4 + 16x^3 + 27x^2 + 22x + 6 = 0$ $\Rightarrow a = 27, \quad b = 22$ **Alternative solution**	 M1 M1 A1 A1		

Question			Answer	Marks	AOs	Guidance
			$f\left(-\frac{3}{2}\right)=\frac{81}{4}-54+\frac{9a}{4}-\frac{3b}{2}+6=0$ $f\left(-\frac{1}{2}\right)=\frac{1}{4}-2+\frac{a}{4}-\frac{b}{2}+6=0$ $a=27$ $b=22$	**M1** **M1** **A1** **A1**		
				[4]		

Question			Answer	Marks	AOs	Guidance
11	(a)	(i)		M1 A1 [2]	1.1 1.1	on a circle centre O form an approximate equilateral triangle B and C must be labelled
11	(a)	(ii)	$z_1 + z_2 + z_3 = z_1\left(1 + e^{2i\pi/3} + e^{4i\pi/3}\right)$ $= z_1 \dfrac{1 - e^{2i\pi}}{1 - e^{2i\pi/3}} = 0$	M1 A1	2.1 2.2a	sum of GP used or other correct method
			Alternative solution $z_1 + z_2 + z_3 =$ $z_1\left[1 + \cos\dfrac{2\pi}{3} + \cos\dfrac{4\pi}{3} + i\left(\sin\dfrac{2\pi}{3} + \sin\dfrac{4\pi}{3}\right)\right]$ $= z_1\left[1 - \dfrac{1}{2} - \dfrac{1}{2} + i\left(\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{2}\right)\right] = 0$	M1 A1		
				[2]		
11	(b)		$z^3 = 8\left(\cos\dfrac{1}{2}\pi + i\sin\dfrac{1}{2}\pi\right)$ $= 8e^{i\pi/2}$ $\|z\| = 2$ $z = 2e^{i\pi/6},\ \ 2e^{5i\pi/6},\ \ 2e^{3i\pi/2}$ $= \sqrt{3} + i,\ \ -\sqrt{3} + i,\ \ -2i$	B1 B1 B1B1 [4]	3.1a 1.1 1.1	Exponential or modulus argument form soi Soi B1B0 if two out of three roots given

Question			Answer	Marks	AOs	Guidance
12			$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x}{4-x^2}y = \frac{1}{4-x^2}$	**B1**	**2.1**	
			IF $\mathrm{e}^{-\int \frac{x}{4-x^2}\,\mathrm{d}x}$	**M1**	**2.1**	Integral must come from an attempt to get $\frac{dy}{dx}$ on its own
			$= \mathrm{e}^{\frac{1}{2}\ln(4-x^2)}$	**M1**	**2.1**	For integrating
			$= \sqrt{4-x^2}$	**A1**	**2.2a**	
			$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{4-x^2}\,y\right) = \frac{1}{\sqrt{4-x^2}}$	**M1**	**2.1**	Multiplying both sides by **their** IF
			$\sqrt{4-x^2}\,y = \int \frac{1}{\sqrt{4-x^2}}\,\mathrm{d}x$			
			$= \arcsin\frac{x}{2} + c$	**A1**	**1.1**	
			$y = \frac{\arcsin\left(\frac{1}{2}x\right) + c}{\sqrt{4-x^2}}$	**M1**	**2.2a**	Rearranging into the form y=, equation must come from an attempt at integration having used IF and include c
			when $x = 0$, $y = 1 \Rightarrow c = 2$	**M1**	**2.1**	Substituting in x = 0, y = 1 to lead to a value of c
			$y = \frac{\arcsin\left(\frac{1}{2}x\right) + 2}{\sqrt{4-x^2}}$	**A1**	**2.2a**	
				[9]		

Question			Answer	Marks	AOs	Guidance
13	(a)		$\overrightarrow{AB} = 6\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$ angle between line and normal is θ where $\cos\theta =$ $\frac{(3\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}).(\boldsymbol{i}-2\boldsymbol{j})}{\sqrt{14}\sqrt{5}} = \frac{-1}{\sqrt{14}\sqrt{5}}$ $\theta = 96.9°$, so angle with plane is 6.9°	B1 M1 A1 A1 [4]	1.1 3.1a 1.1 1.1	Any multiple soi 7° or better 0.1198 rad
13	(b)		eqn of AB is $[\boldsymbol{r} =]4\boldsymbol{i} - \boldsymbol{k} + \lambda(3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ Substituting $x = 4 + 3\lambda, \quad y = 2\lambda, \quad z = -1 - \lambda$: $4 + 3\lambda - 4\lambda = 5 \Rightarrow \lambda = -1$ meets Π_1 at (1, –2, 0) (say C) substituting into Π_2 $2 \times 1 + 3 \times (-2) - 0 = -4$, so C lies on Π_2	B1ft M1 A1 M1 A1 [5]	2.1 1.1 2.2a 3.1a 2.2a	Soi Note may use another position vector as long as it is correct or solving with Π_2
			Alternative method eqn of AB is $[\boldsymbol{r} =]4\boldsymbol{i} - \boldsymbol{k} + \lambda(3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$ Substituting $x = 4 + 3\lambda, \quad y = 2\lambda, \quad z = -1 - \lambda$: $4 + 3\lambda - 4\lambda = 5 \Rightarrow \lambda = -1$ Substituting $x = 4 + 3\lambda, \quad y = 2\lambda, \quad z = -1 - \lambda$: $2(4 + 3\lambda) + 3(2\lambda) - (-1 - \lambda) = -4 \Rightarrow \lambda = -1$ Finding equal λ for both planes (1,-2,0)	 B1 M1 M1 A1 A1 [5]		 soi
13	(c)	(i)	$(\boldsymbol{i} - 2\boldsymbol{j}) \times (2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}) = 2\boldsymbol{i} + \boldsymbol{j} + 7\boldsymbol{k}$	B1 [1]	1.1	
13	(c)	(ii)	$\sqrt{54}$ $\sqrt{5} \times \sqrt{14}\sin\theta = \sqrt{54}$ $\Rightarrow \theta = 61.4°$	B1 M1 A1 [3]	3.1a 1.1 1.1	soi 1.07 rad

Question			Answer	Marks	AOs	Guidance
13	(c)	(iii)	$2\boldsymbol{i}+\boldsymbol{j}+7\boldsymbol{k}$ is direction vector of line of intersection	**B1ft**	**3.1a**	Soi by using in both numerator and denominator in correct method for d
			$\begin{pmatrix}3\\2\\-1\end{pmatrix}\times\begin{pmatrix}2\\1\\7\end{pmatrix}=\begin{pmatrix}15\\-23\\-1\end{pmatrix}$	**M1**	**1.1**	
			$d=\dfrac{\sqrt{15^2+(-23)^2+(-1)^2}}{\sqrt{2^2+1^2+7^2}}$	**M1**	**1.1**	
			$=3.74$	**A1cao** **[4]**	**1.1**	

Question			Answer	Marks	AOs	Guidance
14	(a)		$(3-e^{2i\theta})(3-e^{-2i\theta})=9-3(e^{2i\theta}+e^{-2i\theta})+1=$	M1	1.1	For expanding correctly
			$=10-6\cos 2\theta$	A1 [2]	1.1	
14	(b)		let $S=\sin\theta+\frac{1}{3}\sin 3\theta+\frac{1}{9}\sin 5\theta+\frac{1}{27}\sin 7\theta+\ldots$ and $C=\cos\theta+\frac{1}{3}\cos 3\theta+\frac{1}{9}\cos 5\theta+\frac{1}{27}\cos 7\theta+\ldots$ $C+iS=e^{i\theta}+\frac{1}{3}e^{3i\theta}+\frac{1}{9}e^{5i\theta}+\frac{1}{27}e^{7i\theta}+\ldots$	M1	2.1	At least 2 terms of C + iS soi by correct GP formula
			$=\dfrac{e^{i\theta}}{1-\frac{1}{3}e^{2i\theta}}$	M1 A1	2.1 2.2a	sum to infinity of GP formula for their series (which must be geometric) oe
			$=\dfrac{3e^{i\theta}}{3-e^{2i\theta}}=\dfrac{3e^{i\theta}(3-e^{-2i\theta})}{10-6\cos 2\theta}$	M1*	3.1a	multiply numerator and denominator by $3-e^{-2i\theta}$
			$=\dfrac{9(\cos\theta+i\sin\theta)-3(\cos\theta-i\sin\theta)}{10-6\cos 2\theta}$	M1dep*	2.1	$e^{i\theta}=\cos\theta+i\sin\theta$ used when denominator has been simplified to a real expression
			$S=\dfrac{9\sin\theta+3\sin\theta}{10-6\cos 2\theta}=\dfrac{6\sin\theta}{5-3\cos 2\theta}$	A1 [6]	2.2a	AG

Question			Answer	Marks	AOs	Guidance
15	(a)	(i)	$m\dfrac{d^2x}{dt^2} = -2mx \Rightarrow \dfrac{d^2x}{dt^2} + 2x = 0$	B1 [1]	3.1b	AG
15	(a)	(ii)	Simple harmonic motion	B1 [1]	3.3	
15	(a)	(iii)	$\frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ (s)	B1 [1]	1.2	Accept anything wrt 4.4
15	(a)	(iv)	General solution: $x = A\cos\sqrt{2}\,t + B\sin\sqrt{2}\,t$ $t = 0,\quad x = 2 \Rightarrow A = 2$ $\dfrac{dx}{dt} = -\sqrt{2}A\sin\sqrt{2}\,t + \sqrt{2}B\cos\sqrt{2}\,t$ $t = 0,\quad \dfrac{dx}{dt} = 1 \Rightarrow B = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$ so $x = 2\cos\sqrt{2}\,t + \frac{\sqrt{2}}{2}\sin\sqrt{2}\,t$	B1 B1 M1 A1 [4]	1.1 3.3 1.1 3.3	 Must come from correct GS Must be differentiating a function in terms of cos and sin
15	(a)	(v)	Amplitude $= \sqrt{2^2 + \left(\frac{1}{2}\sqrt{2}\right)^2}$ $= \frac{3\sqrt{2}}{2}$ (m)	M1 A1 [2]	3.4 1.1	 Accept anything wrt 2.1
15	(b)	(i)	$m\frac{d^2x}{dt^2} = -2mx - 2m\frac{dx}{dt}$ $\Rightarrow \dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 2x = 0$	B1 [1]	3.1b	AG
15	(b)	(ii)	Underdamped as $2^2 - 4 \times 1 \times 2 < 0$	B1 [1]	3.5b	Stating complex roots of auxillary equation is acceptable $(-1 \pm i)$

Question			Answer	Marks	AOs	Guidance
15	(b)	(iii)	Auxiliary equation: $\lambda^2 + 2\lambda + 2 = 0$	M1	1.2	
			$\lambda = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$	A1	1.1	soi
			General solution: $x = e^{-t}(A\cos t + B\sin t)$	A1	1.1	
				[3]		
15	(c)	(i)	Particular integral: $x = C\cos 2t + D\sin 2t$	M1	2.1	
			$-4C + 4D + 2C = 2, \quad -4D - 4C + 2D = 0$	M1	3.3	
			$\Rightarrow C = -0.2, \quad D = 0.4$	A1	2.2a	
			$x = e^{-t}(A\cos t + B\sin t) - 0.2\cos 2t + 0.4\sin 2t$			
			$t = 0, \quad x = 2 \Rightarrow A = 2.2$	B1	3.3	Has to come from a correct CF
			$\frac{dx}{dt} = -e^{-t}(A\cos t + B\sin t) + e^{-t}(-A\sin t + B\cos t) + 0.4\sin 2t + 0.8\cos 2t$	M1*	2.1	Must include use of product rule from x = CF + PI
			$t = 0, \frac{dx}{dt} = 1 \Rightarrow 1 = -A + B + 0.8$	M1dep*	3.3	For substitution to lead to an equation in A and B
			$\Rightarrow B = 2.4$			
			$x = e^{-t}(2.2\cos t + 2.4\sin t) - 0.2\cos 2t + 0.4\sin 2t$	A1cao	2.2a	
				[7]		
15	(c)	(ii)	As $t \to \infty, \quad x \to -0.2\cos 2t + 0.4\sin 2t$	M1	3.5a	Dependent on a function containing e^{-kt} and $\cos pt$ and $\sin pt$
			[this is SHM] with period $\frac{2\pi}{2} = \pi \approx 3.14\ s$	A1	3.5a	
				[2]		

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